NAG Toolbox for MATLAB

f07aa

1 Purpose

f07aa computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n matrix and X and B are n by r matrices.

2 Syntax

```
[a, ipiv, b, info] = f07aa(a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07aa uses the LU decomposition with partial pivoting and row interchanges to factor A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda**,$*$) – **double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n coefficient matrix A.

2: **b**(**ldb**,$*$) – **double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The second dimension of the array **a**.

n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: **nrhs_p** ≥ 0.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

5.4 Output Parameters

1: **a**(**lda,**$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

2: **ipiv**$(*)$ **– int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

If **info** ≥ 0, the pivot indices that define the permutation matrix P; at the ith step row i of the matrix was interchanged with row **ipiv**(i). **ipiv**$(i) = i$ indicates a row interchange was not required.

3: **b**(**ldb,**$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

If **info** $= 0$, the n by r solution matrix X.

4: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **nrhs_p**, 3: **a**, 4: **lda**, 5: **ipiv**, 6: **b**, 7: **ldb**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, u_{ii} is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies the equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1}$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

Following the use of f07aa, f07ag can be used to estimate the condition number of A and f07ah can be used to obtain approximate error bounds. Alternatives to f07aa, which return condition and error estimates directly are f04ba and f07ab.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3 + 2n^2r$, where r is the number of right-hand sides.

The complex analogue of this function is f07an.

9 Example

```
a = [1.8, 2.88, 2.05, -0.89;
     5.25, -2.95, -0.95, -3.8;
     1.58, -2.69, -2.9, -1.04;
     -1.11, -0.66, -0.59, 0.8];
b = [9.52;
     24.35;
     0.77;
     -6.22];
[aOut, ipiv, bOut, info] = f07aa(a, b)
```

```
aOut =
    5.2500   -2.9500   -0.9500   -3.8000
    0.3429    3.8914    2.3757    0.4129
    0.3010   -0.4631   -1.5139    0.2948
   -0.2114   -0.3299    0.0047    0.1314
ipiv =
           2
           2
           3
           4
bOut =
    1.0000
   -1.0000
    3.0000
   -5.0000
info =
           0
```